

Practice worksheet

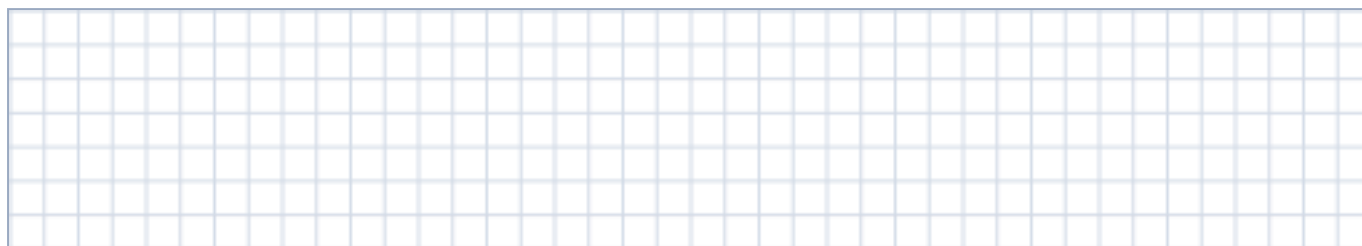
Lesson 6 • Algebraic inequalities II

Name: _____

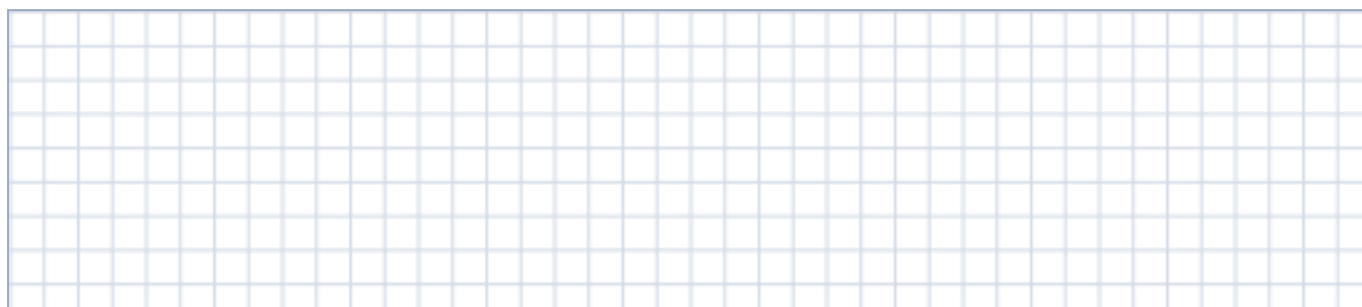
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AM-GM

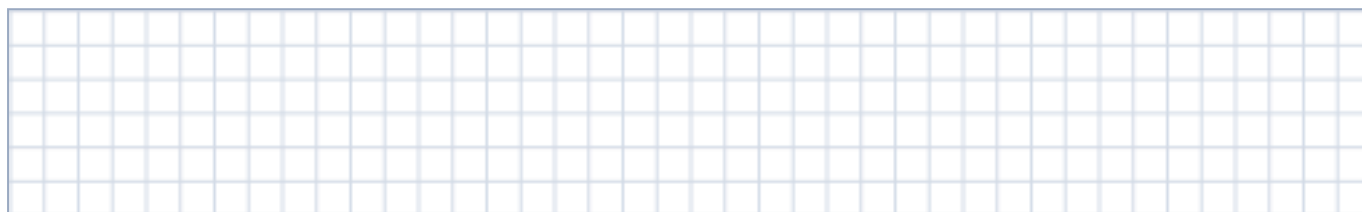
1. Prove that $x + \frac{16}{x} \geq 8$ for $x > 0$, and state when equality holds.



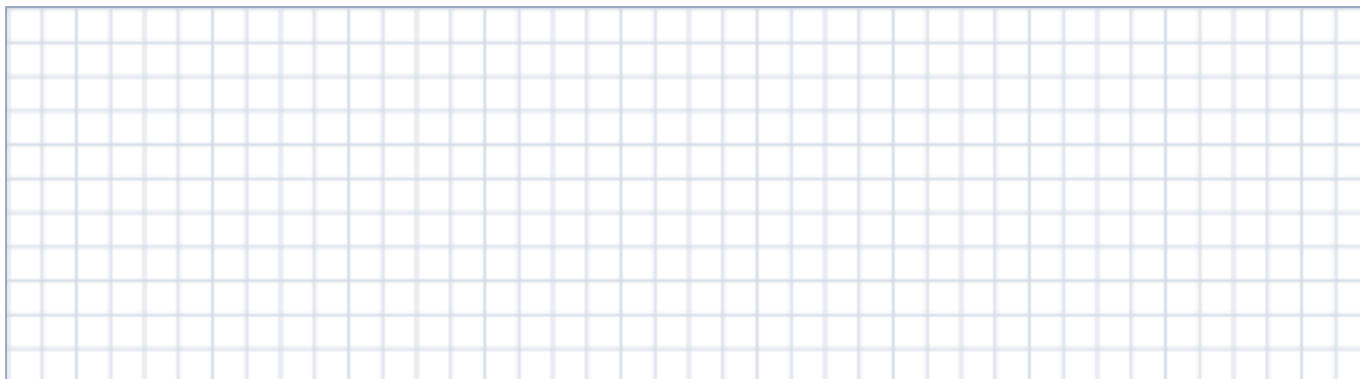
2. Prove that $(a + 1)(b + 1)(a + b) \geq 8ab$ for $a, b > 0$.



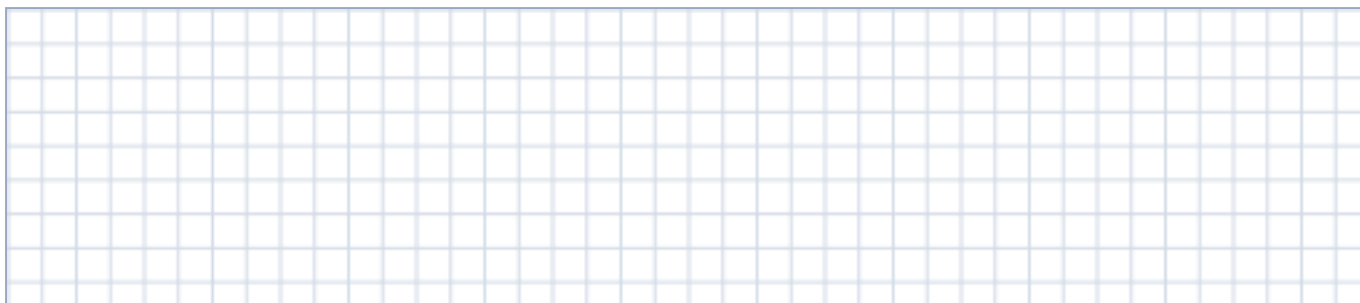
3. Prove that $|a - b| \leq |a| + |b|$ for real a, b .



4. Prove that $||a| - |b|| \leq |a - b|$. (Hint: use $|a| = |(a - b) + b|$ and $|b| = |(b - a) + a|$.)

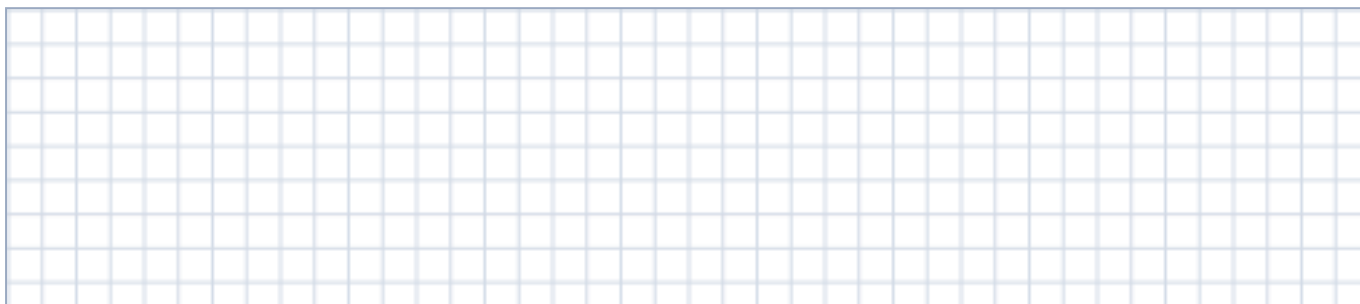


5. Two sides of a triangle are 7 cm and 12 cm. (a) Find the possible lengths of the third side. (b) How many integer lengths are possible? (c) If the triangle is isosceles, find its sides.

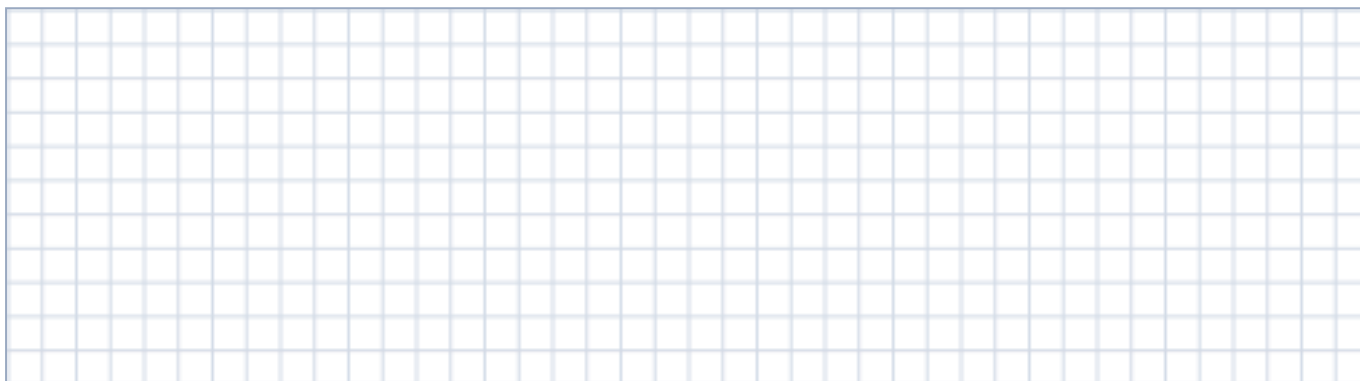


Geometry

6. A rectangle has area 36 cm^2 . Prove that its perimeter is at least 24 cm, and state when it equals 24 cm.



7. A right-angled triangle has hypotenuse 10 cm. Prove that its perimeter is at most $10 + 10\sqrt{2}$ cm. When is this achieved?



Adapted HSC-style

8. (a) Prove that $(a + b) \left(\frac{1}{a} + \frac{1}{b} \right) \geq 4$ for $a, b > 0$. [2] (b) Hence show that if $a + b = 1$, then $\frac{1}{a} + \frac{1}{b} \geq 4$. [1] (c) Hence show that if $a, b > 0$ and $a + b = 1$, then $\frac{1}{a^2} + \frac{1}{b^2} \geq 8$. [2]

