

Practice worksheet

Lesson 5 • Algebraic inequalities I

Name: _____

Date: _____

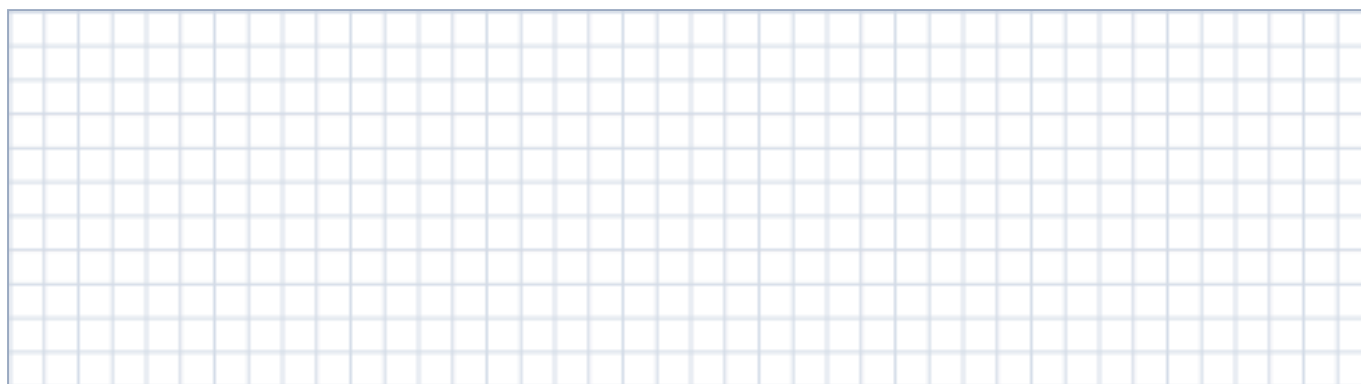
Squares are non-negative

1. Prove each inequality for all real numbers.

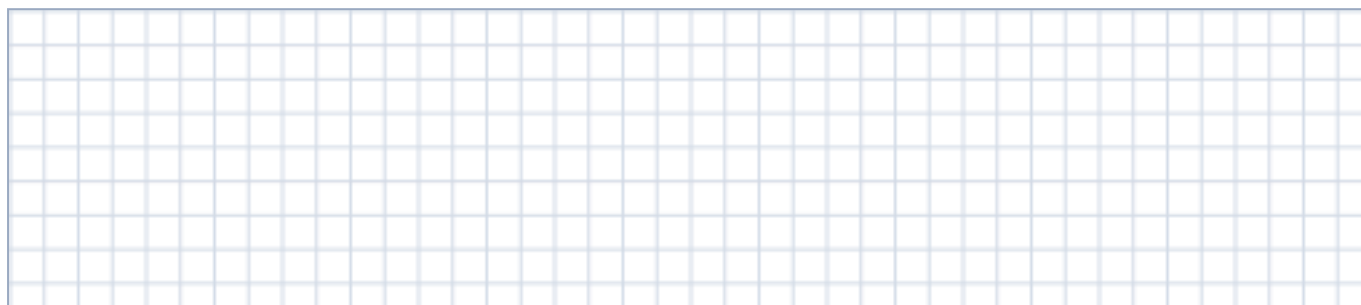
(a) $x^2 - 6x + 10 > 0$

(b) $a^2 + b^2 + 2 \geq 2(a + b)$

(c) $x^2 + y^2 + 1 \geq x + y + xy$



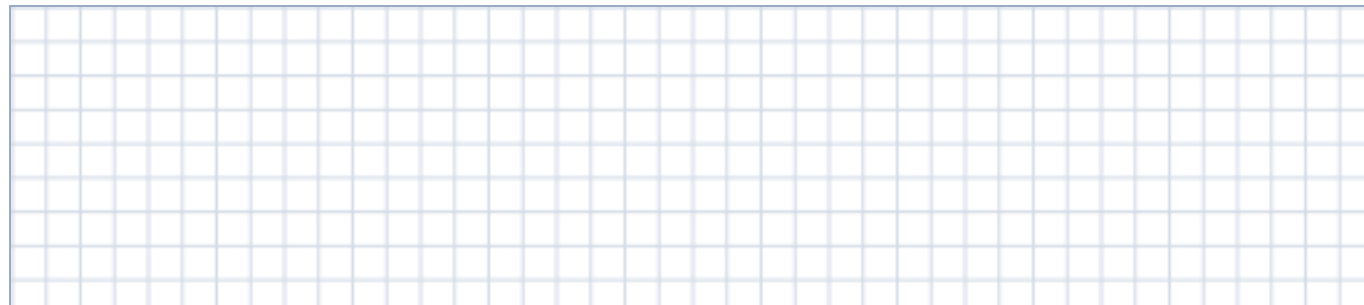
2. Prove that $a^4 + b^4 \geq a^3b + ab^3$ for all real a and b .



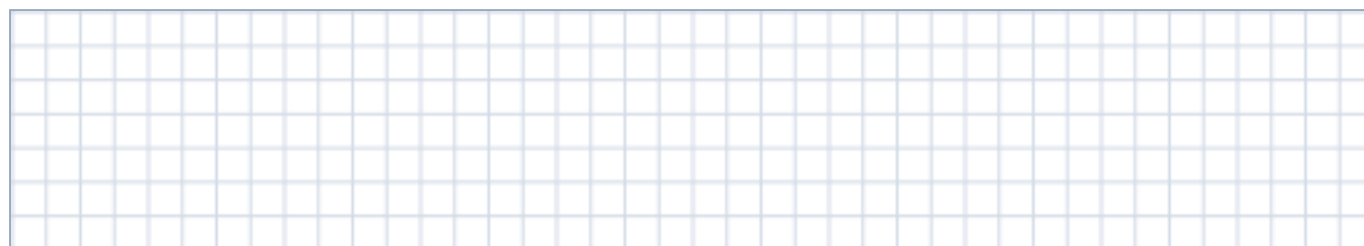
Properties of inequalities

3. Give a counterexample to each statement.

(a) If $a > b$ and $c > d$, then $a - c > b - d$. (b) If $a > b$ and $c > d$, then $\frac{a}{c} > \frac{b}{d}$.



4. Given $a > b > 0$, prove that $\frac{a}{a+1} > \frac{b}{b+1}$.

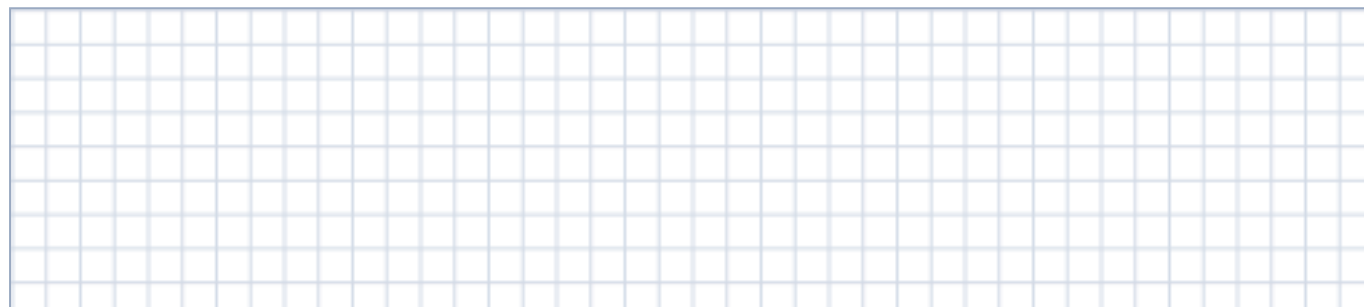


Using a given condition

5. Let $a, b > 0$ with $a + b = 2$. (a) Prove that $ab \leq 1$. (b) Prove that $a^2 + b^2 \geq 2$. (c) Prove that $\frac{1}{a} + \frac{1}{b} \geq 2$.

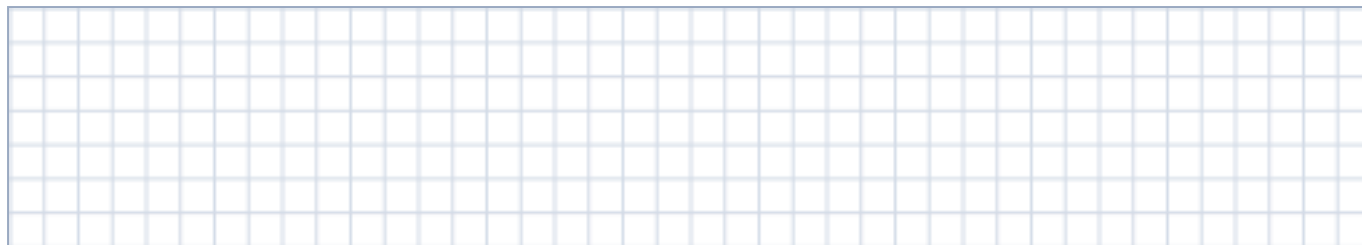


6. For $a, b, c > 0$, prove that $(a + b + c) \left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c} \right) \geq 9$.



Spot the flaw

7. A student "proves" that $x^2 + 1 \geq 2x$ as follows: " $x^2 + 1 \geq 2x \Rightarrow x^2 - 2x + 1 \geq 0 \Rightarrow (x - 1)^2 \geq 0$, which is true, so the claim holds." Explain the flaw and write a correct proof.



Adapted HSC-style

8. (a) Prove that $x^2 + y^2 \geq 2xy$ for all real x, y . [1] (b) Hence show that $(a + b)^2 \leq 2(a^2 + b^2)$ for all real a, b . [1] (c) Hence show that if $a + b + c = 1$, then $a^2 + b^2 + c^2 \geq \frac{1}{3}$. [2]

