MCINTYRE MATHS · MATHEMATICS EXTENSION 2

Practice worksheet

Lesson 2 · Quantifiers, converse and contrapositive

Name: ____________________ Date: ____________

Converse and equivalence

1. Write the converse of each statement. Is it true? If it is, write an iff statement. If not, give a counterexample.

(a) If a number is divisible by 6, then it is divisible by 3.

(b) If $x = 2$, then $x^2 = 4$.

(c) If a triangle is equilateral, then it is isosceles.

(d) If n is odd, then n^2 is odd.

Contrapositive

2. Write the contrapositive of each statement.

(a) If n is divisible by 4, then n is even.

(b) If $a > 0$ and $b > 0$, then $ab > 0$.

(c) If ab is odd, then a and b are both odd.

3. Show that the converse of “If $x > 3$, then $x^2 > 9$” is false, but its contrapositive is true.

Quantifiers

4. Write each statement in symbols, decide whether it is true, and write its negation.

(a) Every integer is a rational number.

(b) Some real number is not equal to its square.

(c) There is an integer whose square is negative.

5. Use a counterexample to show that the statement $\forall x \in \mathbb{R},\ x^2 + x > 0$ is false.

6. Which of these is true? Justify your answer. (a) $\forall x \in \mathbb{R},\ \exists y \in \mathbb{R}$ such that $x + y = 0$. (b) $\exists y \in \mathbb{R}$ such that $\forall x \in \mathbb{R},\ x + y = 0$.

Sufficient and necessary

7. Consider: “If a quadrilateral is a square, then it is a rectangle.” Is the statement true? Name a sufficient condition and a necessary condition. Is the converse true?

Adapted HSC-style

8. **Adapted HSC-style.** Consider the statement: “For all integers n, if n is divisible by 6, then n is divisible by 3.” (a) Write the converse. (b) Is the converse true? Justify your answer. (c) Write the negation of the original statement.