

Practice worksheet

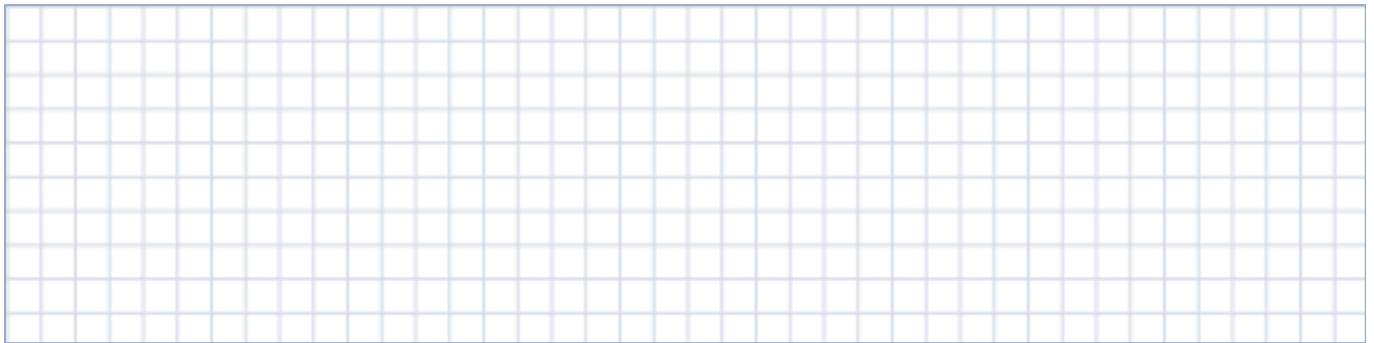
Lesson 7 • Complex numbers as vectors

Name: _____

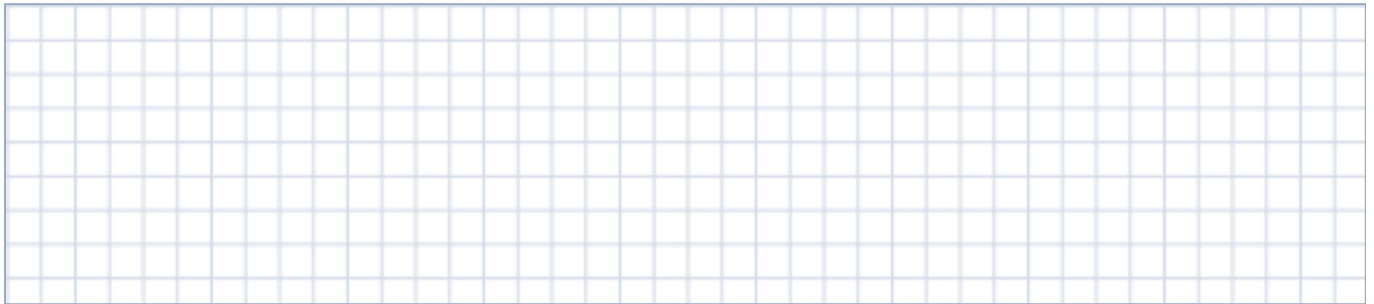
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Vectors and parallelograms

1. P , Q and R represent $1 + 4i$, $-2 + i$ and $3 - 2i$. Find the complex numbers represented by (a) $\overrightarrow{PQ}$, (b) $\overrightarrow{QR}$, (c) $\overrightarrow{RP}$, and show that $\overrightarrow{PQ} + \overrightarrow{QR} + \overrightarrow{RP} = 0$.

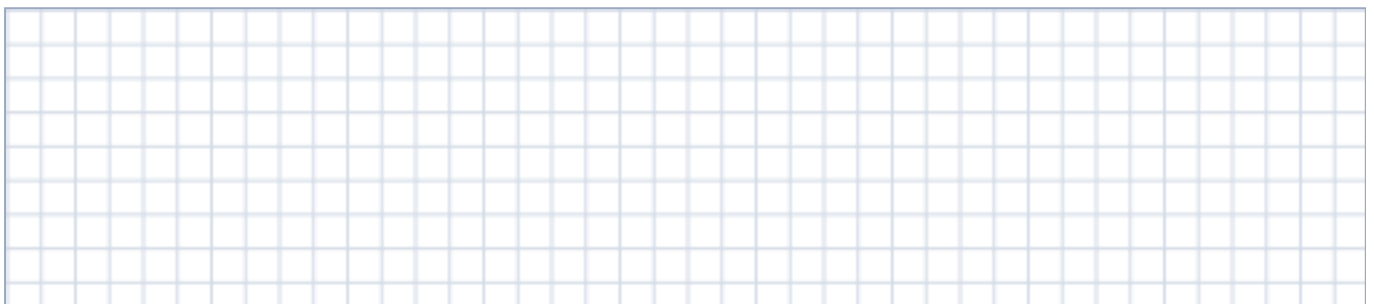


2. $OABC$ is a parallelogram with A at $6 - i$ and C at $-1 + 4i$. Find B , $\overrightarrow{AC}$ and the lengths of the two diagonals.



Distance and direction

3. W is $2 + 3i$ and Z is $5 + 7i$. Find $|z - w|$ and sketch $\overrightarrow{WZ}$ on an Argand diagram. Find $\arg(z - w)$ correct to two decimal places.

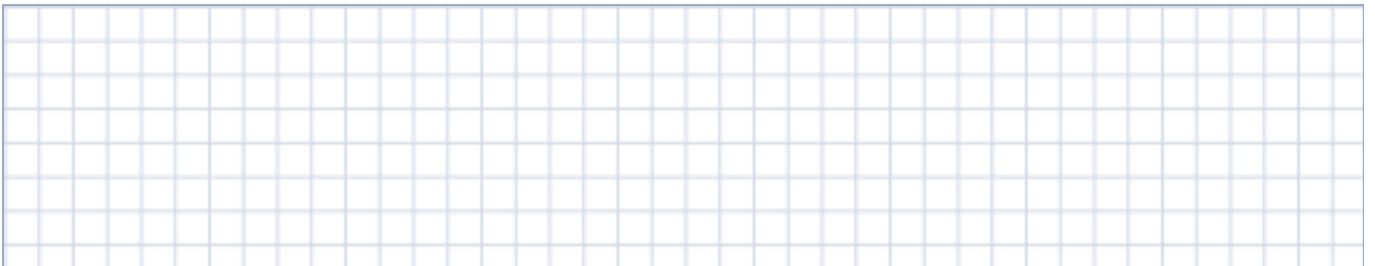


Triangle inequality

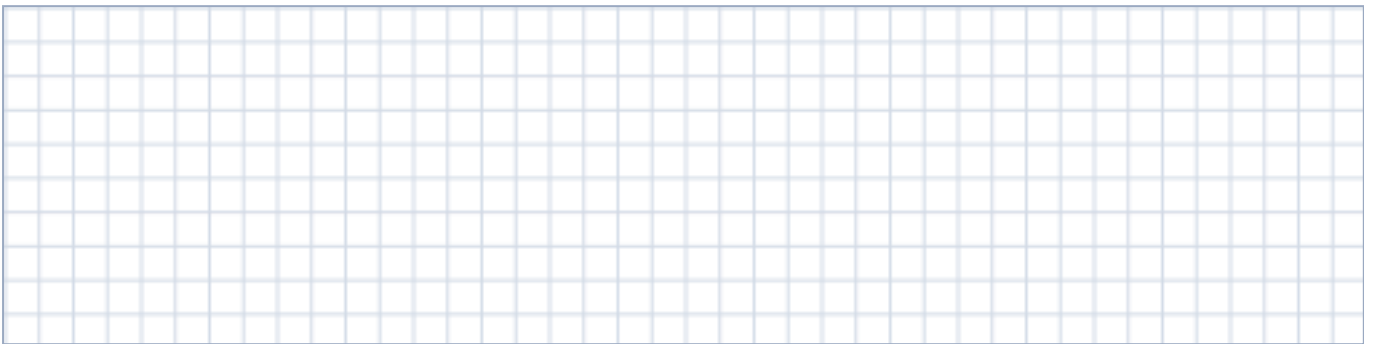
4. Show that for any complex numbers $|z_1 - z_2| \geq ||z_1| - |z_2||$.



5. Given $|z| \leq 2$, find the greatest possible value of $|z^2 + 3z|$ using the triangle inequality, and explain why it is actually attained.



6. Prove that $|z + w|^2 + |z - w|^2 = 2|z|^2 + 2|w|^2$, and interpret it for a parallelogram.



HSC-style

7. Adapted HSC-style. Let $z = \operatorname{cis} \frac{\pi}{3}$ and $w = \operatorname{cis} \frac{\pi}{2}$. (a) Show that $|z + w| = 2 \cos \frac{\pi}{12}$. (b) Hence find $\arg(z + w)$ and explain your answer using a rhombus. (c) By equating real parts, show that $2 \cos \frac{\pi}{12} \cos \frac{5\pi}{12} = \frac{1}{2}$.

